

Patterns

Unit 8 Overview: 7 lessons

Students look for patterns in the addition table and the multiplication table and explain why they work: a row goes up by 1, the sums down a line from top right to bottom left stay the same, the 5s row ends in 0 or 5, and the 4s row is double the 2s row. They predict whether a sum or product is odd or even without working it out, and say which rules are always true. In some states the unit goes on to number patterns: finding a rule and keeping a pattern going, finding a mistake and the number at a position, growing and shrinking shape patterns, and input-output tables.

THE UNIT, DAY BY DAY

Day 1	Pre-Check	What students already know, before the first lesson.
Day 2	8.1 Patterns in the addition table	I can find patterns in the addition table.
Day 3	8.2 Patterns in the times table	I can find patterns in the times table.
Day 4	8.3 Odd and even sums and products	I can tell if a sum or product is odd or even.
Day 5	8.4 Find the rule and keep it going ★	I can find a rule and keep a pattern going.
Day 6	8.5 Pattern mistakes and positions ★	I can find a mistake and the number at a position.
Day 7	8.6 Growing and shrinking shape patterns ★	I can draw and count the steps of a shape pattern.
Day 8	8.7 Input-output tables ★	I can find and use the rule of a table.
Day 9	Unit Review	The review, to go over together.
Day 10	Unit Test	Form A; Form B (other numbers) is for a retest.

★ Taught in 3rd grade in some states. Skip these lessons if yours teaches them in another grade; animalmath.com/standards shows your state's list.

GET READY FOR THE UNIT

- An addition table and a multiplication table (0 to 10) for each student
- Highlighters or crayons in two colors to shade patterns
- Square tiles or pattern blocks for shape patterns
- Whiteboards and dry erase markers
- Chart paper for a class list of patterns and why they work

HOW EACH LESSON GOES

- 1. Warm up** (5 min)
 - 2. Teach** (10 min)
 - 3. Try it together** (10 min)
 - 4. Practice** (15 min)
 - 5. Exit ticket** (5 min): a quick check on the lesson.
- Each lesson page ends with the Animal Math sheets, makers, tools and quizzes for it. "Sheet 3" is the third worksheet of a pack; a color pack prints each sheet again in black and white.

CHECKING AS YOU GO

PRINTABLE	Pre-Check: Day 1, with a key	EXIT TICKET	Exit Tickets: one a lesson, four to a page
PRINTABLE	Unit Review: the day before the test	PRINTABLE	Unit Test: Forms A and B, with keys



Patterns

Lesson 8.1: Patterns in the addition table

- GOAL** Students find patterns in the addition table and explain them: along a row each sum is 1 more, the doubles go up by 2, the sums from top right to bottom left are all the same ($1 + 9$, $2 + 8$, $3 + 7$), and the table matches itself across the doubles because $6 + 3 = 3 + 6$. **About 45 minutes**
- I can find patterns in the addition table.**

THE LESSON

- 1 Warm up** (5 min) Add quickly: $4 + 5$, $4 + 6$, $4 + 7$. What do you notice? Then $9 + 1$, $8 + 2$, $7 + 3$.
- 2 Teach** (10 min) Show the addition table. Shade the 4s row: 4, 5, 6, 7, each 1 more, because the number added goes up by 1. Shade the sums of 10 from top right to bottom left: $1 + 9$, $2 + 8$, $3 + 7$. One number goes up 1 and the other goes down 1, so the sum stays the same. Shade the doubles down the middle: 2, 4, 6, 8, up by 2. Show $6 + 3$ and $3 + 6$ across the doubles.
Say: "When one addend goes up by 1 and the other goes down by 1, the sum stays the same."
- 3 Try it together** (10 min) Pairs fill in a piece of the table on whiteboards (rows 2 to 4, columns 6 to 8), then shade a line of it and say how it changes and why. Share patterns on the class chart.
- 4 Practice** (15 min) Students do sheet 1 of Arithmetic Patterns (the addition table: fill in a piece, shade the sums of 9, rows, columns and diagonals). Early finishers shade every even sum and describe the pattern.
- 5 Exit ticket** (5 min) Students do **Exit Ticket 8.1** and circle a face beside the "I can" line. On question 1, look for 12, 12 and 14 in the blanks. On question 2, look for 7, 9 and 11 and "Up by 2"; "Up by 1" means the student looked at only one addend.

Need help? Build each sum with two colors of cubes and move one cube from one color to the other.

Ready for more? Find a line in the table where the sums go down by 2, and explain why.

GET READY

- An addition table for each student
- Highlighters in two colors
- Whiteboards and markers

WORDS TO USE

row: numbers that go across

column: numbers that go down

diagonal: a slanted line from corner to corner

turn-around fact: the same numbers added in the other order, like $6 + 3$ and $3 + 6$

ASK

- How does the sum change when you move one place right?
- Why do these sums stay the same?
- Where is $3 + 6$ in the table? Where is $6 + 3$?

WATCH FOR

Reads a row and a column mixed up. Put a finger on each: the row number on the side, the column number on top.

Says a diagonal "goes up by 1". Write the sums down the line and find each jump.

ON ANIMAL MATH

PRINTABLE

Arithmetic Patterns: sheet 1

EXIT TICKET

3rd Grade Patterns Exit Tickets: Exit Ticket 8.1



Patterns

Lesson 8.2: Patterns in the times table

GOAL

Students find patterns in the times table and explain them: the 5s row ends in 0 or 5, the 10s row ends in 0, the 2s row is all even, and the 4s row is double the 2s row because 4 groups is 2 groups and 2 groups more.

About 45 minutes

I can find patterns in the times table.

THE LESSON

- 1 Warm up** (5 min) Count by 5s to 50 and by 10s to 100. What do the numbers end in?
- 2 Teach** (10 min) Shade the 5s row: 5, 10, 15, 20, every number ends in 0 or 5. Shade the 2s row: all even. Put the 2s and 4s rows side by side: $2 \times 7 = 14$ and $4 \times 7 = 28$, $2 \times 9 = 18$ and $4 \times 9 = 36$. Each 4s number is double the 2s number above it, because 4 groups is 2 groups and 2 more groups. Then the 9s row: 18, 36, 45, the digits add to 9.
Say: "A row of the times table is skip counting. Doubling the number of groups doubles the product."
- 3 Try it together** (10 min) Pairs use the map page of Pattern Explorer to find a row that is double another row and check three columns, then tell the class why it works.
- 4 Practice** (15 min) Students do the stops on sheets 1 to 4 of Pattern Explorer: Multiplication Patterns, then sheet 4 of Arithmetic Patterns (multiples and their digits). Early finishers do sheet 5 (who is right?).
- 5 Exit ticket** (5 min) Students do **Exit Ticket 8.2** and circle a face beside the "I can" line. On question 1, look for 24, 40 and 56 and "double"; "4 more" or "8 more" means the student did not check a column. On question 2, look for 27, 54 and 81 circled.

Need help? Build 2 groups and 4 groups of the same number with cubes and compare.

Ready for more? Which row is triple the 2s row? Which row is the 2s row and the 3s row added together? Check on the chart.

GET READY

- A multiplication table for each student
- Highlighters in two colors
- Whiteboards and markers

WORDS TO USE

multiple: a number in a times table row, like 15 in the 5s row

double: two times as many

even: a number that splits into two equal groups

ASK

- What do all the numbers in this row have in common?
- How does this row compare to that row?
- Why is the 4s row double the 2s row?

WATCH FOR

Circles 62 as a 9s number. Count by 9s on the chart: 63, 72. Is 62 there?

Says the 4s row is "2 more" than the 2s row. Check another column: $2 \times 9 = 18$, $4 \times 9 = 36$. Is that 2 more?

ON ANIMAL MATH

PRINTABLE

Pattern Explorer: Multiplication Patterns: sheets 1-4

PRINTABLE

Arithmetic Patterns: sheets 4-5

TOOL

Multiplication Chart: shade two rows to compare

EXIT TICKET

3rd Grade Patterns Exit Tickets: Exit Ticket 8.2

Patterns

Lesson 8.3: Odd and even sums and products

GOAL Students predict whether a sum or product is odd or even without working it out, and explain why: odd + odd is even because the two extra ones make a pair, and a product with an even factor is even.

About 45 minutes

I can tell if a sum or product is odd or even.

THE LESSON

- 1 Warm up** (5 min) Make pairs with 7 counters and with 8 counters. Which has one left over? Is 23 odd or even? Is 46?
- 2 Teach** (10 min) Show $7 + 5$ with counters in pairs: each has one left over, and the two extras make a pair, so 12 is even. Then $6 + 3$: only one extra, so 9 is odd. Write the rules. Products: $4 \times 7 = 28$. 4 groups pair up, so an even factor makes the product even. $3 \times 5 = 15$: odd groups of an odd number leave one over, so odd. Predict: $33 + 25$ is even, $18 + 27$ is odd, 6×9 is even, 7×9 is odd.
Say: "I look at the ones digits. Odd plus odd makes a pair, so the sum is even."
- 3 Try it together** (10 min) Pairs sort cards with sums and products into odd and even without working them out, then check two of them by adding or multiplying. Write "always true" sentences on the class chart.
- 4 Practice** (15 min) Students do sheet 2 of Arithmetic Patterns (odd and even sums) and sheet 3 (odd and even products).
- 5 Exit ticket** (5 min) Students do **Exit Ticket 8.3** and circle a face beside the "I can" line. On question 1, look for odd for $46 + 29$ and even for 3×6 . On question 2, "Even + odd is always odd" should be circled.

Need help? Start with sheet 2 of Odd and Even Numbers to 100 (a review), then use counters in pairs.

Ready for more? Is the sum of three odd numbers odd or even? Explain with pairs.

ON ANIMAL MATH

- PRINTABLE** **Arithmetic Patterns:** sheets 2-3
- PRINTABLE** **Odd and Even Numbers to 100:** sheet 2 (review)
- MAKER** **Odd & Even Numbers:** review: Mode Write Odd or Even, Number Range Within 100
- TOOL** **Multiplication Chart:** shade the odd products
- QUIZ** **Odd, Even and Arrays Quiz:** the Odd or Even level
- EXIT TICKET** **3rd Grade Patterns Exit Tickets:** Exit Ticket 8.3

GET READY

- Counters or cubes
- A multiplication table
- Whiteboards and markers

WORDS TO USE

odd: a number with one left over when you make pairs

even: a number that makes pairs with none left

predict: say what will happen before you work it out

ASK

- Is each number odd or even?
- Do the left over ones make a pair?
- Is that true for every number? Try another.

WATCH FOR

Looks at the tens digit to decide odd or even. Only the ones digit matters: cover the tens.

Thinks odd + odd is odd. Build it with pairs: the two leftover ones make a pair.

Thinks odd $\times$ even is odd. An even number of groups always pairs up.

Patterns

Lesson 8.4: Find the rule and keep it going

GOAL Students find the rule of a number pattern (add, subtract, multiply or divide by a number) by checking every jump, fill in the missing numbers, and keep the pattern going: ★ Taught in 3rd grade in some states

About 45 minutes

I can find a rule and keep a pattern going.

THE LESSON

- 1 Warm up** (5 min) What comes next: 10, 20, 30; 45, 40, 35; 2, 4, 6? Say each rule.
- 2 Teach** (10 min) Write 5, 11, 17, 23, ____, ____. Find each jump: + 6, + 6, + 6. The rule is add 6: 29, 35. Then write 5, 10, 20. Is it add 5? $5 + 5 = 10$, but $10 + 5$ is not 20. Is it times 2? $5 \times 2 = 10$ and $10 \times 2 = 20$. The rule must work for every jump. Then 60, 55, 50: subtract 5.
Say: "I find the first jump, then check it on every jump. A rule has to work all the way."
- 3 Try it together** (10 min) Pairs get pattern cards with two blanks (like 12, 6, 3, a pattern that divides by 2), find the rule, check every jump, and fill in the blanks. Then they make a pattern for a partner.
- 4 Practice** (15 min) Students do sheets 1 to 4 of Complete the Pattern (sheets 5 and 6 for support, 7 and 8 as a challenge), then sheets 1 and 2 of Pattern Detective. Early finishers do sheet 1 of Missing Output: Code Number Patterns.
- 5 Exit ticket** (5 min) Students do **Exit Ticket 8.4** and circle a face beside the "I can" line. On question 1, look for 56 and 40 and "Subtract 8". On question 2, look for 3 and "Divide by 3"; "Subtract 18" means the student checked only the first jump.

Need help? Write the jumps between the numbers above each pair before finding the rule.

Ready for more? Make a pattern where two rules fit the first jump but only one fits all of them.

GET READY

- Whiteboards and markers
- A hundreds chart
- Pattern cards with blanks

WORDS TO USE

pattern: numbers that follow a rule

rule: what you do to get from one number to the next

jump: the change from one number to the next

ASK

- What is the jump from the first number to the next?
- Does that rule work for every jump?
- Are the numbers getting bigger or smaller? What does that tell you?

WATCH FOR

Uses only the first jump (5, 10 is add 5). Check the rule on every jump before writing it.

Writes a rule like "goes up". Name the operation and the number: add 6.

ON ANIMAL MATH

PRINTABLE

Complete the Pattern: Missing Numbers and Rules: sheets 1-4 (5-6 support, 7-8 challenge)

PRINTABLE

Pattern Detective: Find the Number Pattern Rule: sheets 1-2

PRINTABLE

Missing Output: Code Number Patterns: sheet 1

MAKER

Number Patterns: Pattern Type Mixed, Difficulty Hard

EXIT TICKET

3rd Grade Patterns Exit Tickets: Exit Ticket 8.4

Patterns

Lesson 8.5: Pattern mistakes and positions

GOAL Students find the one number that breaks a pattern's rule and fix it (4, 10, 16, 21, 28 should have 22), and find the number at a later position by keeping the pattern going (start at 3, add 4: the 10th number is 39).

About 45 minutes

★ Taught in 3rd grade in some states

I can find a mistake and the number at a position.

THE LESSON

- 1 Warm up** (5 min) Line up five students with number cards 7, 14, 21, 28, 35. Ask: what is the 3rd number? What would the 6th be?
- 2 Teach** (10 min) Write 4, 10, 16, 21, 28. The jumps are + 6, + 6, + 5, + 7. One number is wrong: 21 should be 22, then every jump is + 6. Then a position: start at 3 and add 4 each time. Write the positions 1st to 10th over the numbers: 3, 7, 11, 15 ... the 10th is 39. Or think: 9 jumps of 4 after the 3, so $3 + 36 = 39$.
Say: "I check every jump. The number that breaks the rule is the mistake."
- 3 Try it together** (10 min) Pairs fix the mistake in 50, 45, 41, 35, 30 and find the 8th number of a pattern that starts at 6 and adds 3. Then solve a story: a plant is 5 centimeters tall and grows 3 centimeters a week; how tall after 4 weeks?
- 4 Practice** (15 min) Students do sheets 1 to 4 of Find the Mistake in Number Patterns, then sheets 1 and 2 of Number Patterns: Position and Value. Early finishers do sheet 1 of Number Pattern Word Problems.
- 5 Exit ticket** (5 min) Students do **Exit Ticket 8.5** and circle a face beside the "I can" line. On question 1, 27 should be circled and fixed to 26. On question 2, look for 23, 28, 33 and 38 in the 7th position; 43 means the student counted one jump too many.

Need help? Write each jump above the pattern before looking for the mistake.

Ready for more? Find the position of 51 in the pattern that starts at 3 and adds 4.

GET READY

- Whiteboards and markers
- Number cards to line up as a track
- A hundreds chart

WORDS TO USE

position: where a number is in the pattern: 1st, 2nd, 3rd

mistake: a number that does not follow the rule

value: the number at a position

ASK

- Which jump is different?
- If you change that number, do all the jumps match?
- How many jumps are there from the 1st position to the 10th?

WATCH FOR

Circles the number after the mistake, where a jump looks wrong too. Try fixing each number and check every jump.

Counts 10 jumps to the 10th position. The 1st number needs no jump: from 1st to 10th is 9 jumps.

ON ANIMAL MATH

PRINTABLE Find the Mistake in Number Patterns: sheets 1-4

PRINTABLE Number Patterns: Position and Value: sheets 1-2

PRINTABLE Number Pattern Word Problems: sheet 1

EXIT TICKET 3rd Grade Patterns Exit Tickets: Exit Ticket 8.5

Patterns

Lesson 8.6: Growing and shrinking shape patterns

GOAL Students build and draw the next step of a shape pattern that grows or shrinks by the same number each step, count the squares, and predict a later step: an H grows 3, 7, 11, ★ Taught in 3rd grade in some states

I can draw and count the steps of a shape pattern.

THE LESSON

- 1 Warm up** (5 min) Build a row of 3 tiles, then 6, then 9. How many will the next row have? What is the rule?
- 2 Teach** (10 min) Build an H: step 1 is 3 tiles in a row, step 2 adds a tile above and below each end (7), step 3 adds one more on each of the 4 ends (11). It grows by 4 each step, one tile on each end. Step 4 has 15. Make a table: step and squares. Then a shrinking pattern: 20 squares, then 17, then 14. It loses 3 each step, so step 4 has 11.
Say: "I look at what is added each step and where it goes. Then I can draw the next step and count."
- 3 Try it together** (10 min) Pairs build a pattern that grows by 3 with tiles (3, 6, 9), draw step 4 on grid paper, and fill in a table to step 6. Then they make a shrinking pattern for another pair.
- 4 Practice** (15 min) Students do sheet 1 of Growing Shape Patterns (draw step 4), sheet 2 (count and fill a table) and sheet 3 (shrinking patterns). Early finishers do sheet 4 (predict step 10).
- 5 Exit ticket** (5 min) Students do **Exit Ticket 8.6** and circle a face beside the "I can" line. On question 1, step 4 is a row of 3 on two legs of 4 (11 squares); 10 means only one leg grew. On question 2, look for 6 squares (18, 14, 10, 6).

Need help? Build every step with tiles before drawing it.

Ready for more? Make a growing pattern of your own and write a rule for step 10.

GET READY

- Square tiles or pattern blocks for each pair
- Grid paper
- Whiteboards and markers

WORDS TO USE

step: one picture in the pattern: step 1, step 2

grow: get bigger by the same number each step

shrink: get smaller by the same number each step

ASK

- What changes from one step to the next?
- How many squares are added or taken away each time?
- How could you find step 10 without drawing it?

WATCH FOR

Adds the new squares in the wrong place. Build the step with tiles and compare it to the last step.

Counts a corner square twice. Touch each square once as you count, or count by rows.

Takes away the rule one time too many for step 4. Step 1 is the start: step 4 is three jumps later.

ON ANIMAL MATH

PRINTABLE

Growing Shape Patterns: sheets 1-3 (sheet 4 to stretch)

TOOL

Pattern Blocks: build each step of a growing pattern

EXIT TICKET

3rd Grade Patterns Exit Tickets: Exit Ticket 8.6

Patterns

Lesson 8.7: Input-output tables

GOAL Students use a rule to fill in an input-output table (the rule add 6: 3 makes 9), find the rule from the full pairs (2 makes 10 and 7 makes 35: times 5), and work backward to a missing input.

About 45 minutes

★ Taught in 3rd grade in some states

I can find and use the rule of a table.

THE LESSON

- 1 Warm up** (5 min) Play the machine: you whisper a rule (add 6) and the class calls out inputs. Say each output until someone guesses the rule.
- 2 Teach** (10 min) Draw a table with the rule add 6: 3 makes 9, 10 makes 16. Then a table with no rule: 2 makes 10, 7 makes 35. Is it add 8? $7 + 8$ is 15, not 35. Is it multiply by 5? $2 \times 5 = 10$ and $7 \times 5 = 35$, so the rule is multiply by 5. One pair is not enough: 3 makes 6 could be add 3 or multiply by 2. To find a missing input, work backward: out 40 with multiply by 5 means in 8.
Say: "The rule has to work for every pair. To go backward, I do the opposite."
- 3 Try it together** (10 min) Pairs make a table for a partner with a secret rule, three full pairs and two blanks. The partner finds the rule, checks it on every pair and fills in the blanks.
- 4 Practice** (15 min) Students do sheet 1 of Input and Output Tables (function machines), sheet 2 (finish the tables), sheet 3 (find the rule) and sheet 4 (missing ins and outs). Early finishers do sheet 5 (word problems).
- 5 Exit ticket** (5 min) Students do **Exit Ticket 8.7** and circle a face beside the "I can" line. On question 1, look for Multiply by 3, outs 6 and 27 and the input 7. On question 2, look for 15 and 26, and 9 in the In row (work backward, $4 + 5$); $4 - 5$ or a blank means the student used the rule forward.

Ready for more? Make a table where two rules fit one pair but only one fits all the pairs.

GET READY

- A box drawn as a "function machine" on the board
- Whiteboards and markers
- Tables with In and Out rows

WORDS TO USE

input: the number that goes in

output: the number that comes out

rule: what the machine does to every input

ASK

- What happens to every input?
- Does your rule work for every pair?
- How do you find a missing input?

WATCH FOR

Finds a rule that fits only the first pair. Check the rule on every full pair.

Uses the rule forward to find a missing input. Do the opposite: subtract instead of add, divide instead of multiply.

Need help? Act out the machine with counters: put them in, do the rule, count what comes out.

ON ANIMAL MATH

PRINTABLE

Input and Output Tables: sheets 1–5

EXIT TICKET

3rd Grade Patterns Exit Tickets: Exit Ticket 8.7