

Problems and Patterns

Unit 6 Overview: 8 lessons

Students solve stories in two and more steps with all four operations: they draw strip diagrams, write one equation with a letter for the unknown, and estimate to check that an answer is reasonable. Then they make number and shape patterns from a rule and look for what the rule does not say, such as odd and even numbers taking turns, and explain why it happens. In some states the unit also covers the equal sign with all four operations ($=$ and $\neq$), a letter in any place of an equation, input-output tables, and rules that give a value from its position (lessons 6.3, 6.4, 6.7 and 6.8). Ask “what does the letter stand for?” and “why does the pattern do that?” every day.

THE UNIT, DAY BY DAY

Day 1	Pre-Check	What students already know, before the first lesson.
Day 2	6.1 Two-step stories with a letter	I can write an equation with a letter for a two-step story.
Day 3	6.2 Multi-step stories; estimate to check	I can solve a multi-step story and estimate to check.
Day 4	6.3 True or false with $=$ and $\neq$ ★	I can tell if two sides are equal or not equal.
Day 5	6.4 A letter in any place ★	I can find the value of a letter in an equation.
Day 6	6.5 Number patterns from a rule	I can make a number pattern from a rule.
Day 7	6.6 Shape patterns from a rule	I can make a shape pattern from a rule.
Day 8	6.7 Input-output tables ★	I can find the rule of a table and use it.
Day 9	6.8 Rules by position ★	I can write a rule for a value from its position.
Day 10	Unit Review	The review, to go over together.
Day 11	Unit Test	Form A; Form B (other numbers) is for a retest.

★ Taught in 4th grade in some states. Skip these lessons if yours teaches them in another grade; animalmath.com/standards shows your state's list.

GET READY FOR THE UNIT

- Story cards for two-step and multi-step stories, and strip diagram paper
- Square tiles and pattern blocks for shape patterns
- Hundreds charts and colored pencils for number patterns
- Whiteboards and dry erase markers, and sticky notes
- Chart paper for anchor charts: strip diagrams, $=$ and $\neq$, features of patterns, find the rule

HOW EACH LESSON GOES

- 1. Warm up** (5 min)
 - 2. Teach** (10 min)
 - 3. Try it together** (10 min)
 - 4. Practice** (15 min)
 - 5. Exit ticket** (5 min): a quick check on the lesson.
- Each lesson page ends with the Animal Math sheets, makers, tools and quizzes for it. “Sheet 3” is the third worksheet of a pack; a color pack prints each sheet again in black and white.

CHECKING AS YOU GO

PRINTABLE	Pre-Check: Day 1, with a key	EXIT TICKET	Exit Tickets: one a lesson, four to a page
PRINTABLE	Unit Review: the day before the test	PRINTABLE	Unit Test: Forms A and B, with keys



Problems and Patterns

Lesson 6.1: Two-step stories with a letter

GOAL Students solve two-step stories: they draw a strip diagram, write one equation with a letter for the unknown, and solve it one step at a time in the order the story happens.

About 45 minutes

I can write an equation with a letter for a two-step story.

THE LESSON

- 1 Warm up** (5 min) Solve in two steps: 3 bags of 8 apples, then 5 are eaten. How many are left?
- 2 Teach** (10 min) Read: Kai has 3 packs of 25 cards. He gives away 18. How many are left? Draw 3 bars of 25, and under them 18 and c. Write one equation: $3 \times 25 - 18 = c$. Multiply first, then subtract: $75 - 18 = 57$, so $c = 57$ cards. Multiplying and dividing come before adding and subtracting even when they are written second: in $200 - 4 \times 25 = c$, $4 \times 25 = 100$ comes first, so $c = 100$. When the story adds first, parentheses show it: 42 red beads and 30 blue beads are shared by 6 friends, so $(42 + 30) \div 6 = b$ and $b = 12$ beads each.
Say: "I draw the story and write one equation with a letter. I multiply or divide first, unless parentheses tell me to start somewhere else."
- 3 Try it together** (10 min) Pairs draw strip diagrams and write equations for two story cards: a store has 5 boxes of 36 pens and sells 64 ($5 \times 36 - 64 = p$); a coach runs 90 meters, then 4 runners each run 220 meters ($90 + 4 \times 220 = m$, so 970 meters).
- 4 Practice** (15 min) Students do sheets 1-3 of Multi-Step Word Problems (adding and subtracting, then multiplying and dividing, then the equation that matches). For a step back, sheet 2 of Two-Step Word Problems for 3rd Grade.
- 5 Exit ticket** (5 min) Students do **Exit Ticket 6.1** and circle a face beside the "I can" line. On question 1, look for an equation like $4 \times 32 - 47 = k$ and $k = 81$. On question 2, look for $120 - 5 \times 14 = c$ and 50 cards; 1,610 means $120 - 5$ was done first.

Ready for more? Write a two-step story for $6 \times 15 + 35 = t$ and solve it.

GET READY

- Story cards
- Strip diagram paper or grid paper
- Whiteboards and markers

WORDS TO USE

strip diagram: bars that show the parts of a story

equation: a number sentence with an equal sign

letter: stands for the number we want to find

ASK

- What do the bars show?
- Which step comes first in the story?
- What does your letter stand for?

WATCH FOR

Works left to right in $200 - 4 \times 25$ and gets 4,900. Multiplying comes first: $4 \times 25 = 100$, then $200 - 100 = 100$.

Writes two answers and no equation. Put both steps in one equation with a letter for the answer.

Need help? Act the story out with cubes in cups before drawing the bars.

ON ANIMAL MATH

PRINTABLE

Multi-Step Word Problems: sheets 1-3

PRINTABLE

Two-Step Word Problems for 3rd Grade: sheet 2 (3rd grade), a step back

MAKER

Word Problems: Problem Type Two Steps, Number Range 100-300 (adding and subtracting only)

QUIZ

Word Problems Quiz: the Two Steps, All Four level

EXIT TICKET

4th Grade Multi-Step Problems and Patterns Exit Tickets: Exit Ticket 6.1

Problems and Patterns

Lesson 6.2: Multi-step stories; estimate to check

GOAL Students solve stories with three or more steps and all four operations, one step at a time with each answer labeled, and estimate to check that the answer is reasonable.

About 45 minutes

I can solve a multi-step story and estimate to check.

THE LESSON

- 1 Warm up** (5 min) Round to the nearest hundred and estimate:
 $386 + 417 - 209$.
- 2 Teach** (10 min) Read: A school orders 8 boxes of 144 crayons. It keeps 252 and shares the rest equally among 6 classes. How many does each class get? Step 1: $8 \times 144 = 1,152$ crayons. Step 2: $1,152 - 252 = 900$ to share. Step 3: $900 \div 6 = 150$ for each class. Estimate: $8 \times 150 = 1,200$, and $1,200 - 250 = 950$ to share. $6 \times 150 = 900$ is close to 950, so 150 is reasonable.
Say: "I write one step at a time and label what each answer means. Then I estimate to see if my answer makes sense."
- 3 Try it together** (10 min) Pairs solve: 3 classes of 28 and 2 classes of 31 go on a trip in vans that hold 9. How many vans? ($84 + 62 = 146$, and $146 \div 9 = 16 \text{ R } 2$, so 17 vans.) Then they estimate: about 150 students, about 15 vans, so 17 makes sense.
- 4 Practice** (15 min) Students do sheets 4-5 of Multi-Step Word Problems (estimate or exact, then stories with three steps or more).
- 5 Exit ticket** (5 min) Students do **Exit Ticket 6.2** and circle a face beside the "I can" line. On question 1, look for three steps: 216, 141, then 47 apples; 72 means the 75 sold was never taken away. On question 2, look for $500 + 400 - 200 = 700$.

Need help? Write each step on its own line with a label, like "crayons in all".

Ready for more? Write a three-step story that uses $\times$, $-$ and $\div$, and trade with a partner.

GET READY

- Story cards with three steps
- Whiteboards and markers
- Grid paper

WORDS TO USE

multi-step: a story that takes more than two steps

estimate: a close answer from rounded numbers

reasonable: close to the estimate, so it makes sense

ASK

- What happens first in the story?
- What does each step's answer mean?
- Is your answer close to your estimate?

WATCH FOR

Uses the numbers in the order they appear. Draw the story first and decide which step comes first.

Forgets a step. Label each answer and ask what the story still needs.

ON ANIMAL MATH

PRINTABLE

Multi-Step Word Problems: sheets 4-5

QUIZ

Word Problems Quiz: the Multi-Step, Larger Numbers level

EXIT TICKET

4th Grade Multi-Step Problems and Patterns Exit Tickets: Exit Ticket 6.2

Problems and Patterns

Lesson 6.3: True or false with = and $\neq$

GOAL Students decide whether an equation is true by finding the value of each side, with all four operations, and write = or $\neq$ between two expressions: the equal sign means both sides have the same value, not “the answer comes next”.

About 45 minutes

★ Taught in 4th grade in some states

I can tell if two sides are equal or not equal.

THE LESSON

- 1 Warm up** (5 min) Is $7 + 5 = 12 + 3$ true? Many say yes. Find the value of each side: 12 and 15.
- 2 Teach** (10 min) Write $6 \times 50 = 400 - 100$. Left side: 300. Right side: 300. Same value, so it is true. Then $96 \div 3$ and $30 + 3$: 32 and 33, so write $96 \div 3 \neq 30 + 3$. Last, $8 \times 7 = 56 + 4$ is false: the right side is 60. The equal sign does not mean “write the answer next”.
Say: “I find the value of each side. If they are the same, I write =. If not, I write $\neq$.”
- 3 Try it together** (10 min) Pairs sort expression cards into equal pairs: $120 \div 4$ and 15×2 , $64 + 36$ and 4×25 , 7×40 and $320 - 40$. Then each writes one true and one false equation for a partner to check.
- 4 Practice** (15 min) Students do sheets 1-2 of Equations with All Four Operations (is it true?, then = or $\neq$), then sheet 2 of Variables and Equations. For a step back, sheet 2 of True or False: Multiplication & Division keeps to the facts.
- 5 Exit ticket** (5 min) Students do **Exit Ticket 6.3** and circle a face beside the “I can” line. On question 1, look for = (both sides 420) and $\neq$ (21 and 22). On question 2, look for True (450) and False (90 and 99); circling True for the second means only one side was worked out.

Ready for more? Write four expressions with a value of 240, one for each operation.

GET READY

- Expression cards
- Whiteboards and markers
- A pan balance, if there is one

WORDS TO USE

equal sign: both sides have the same value

not equal ($\neq$): the two sides have different values

expression: numbers and signs with no equal sign, like 6×50

ASK

- What is the value of each side?
- Can a true equation have operations on both sides?
- How do you know they are not equal?

WATCH FOR

Calls $8 \times 7 = 56 + 4$ true because 56 comes next. Find the value of both sides: 56 and 60.

Works out only one side. Write the value under each side before choosing the sign.

Need help? Draw a pan balance: each side is a pan, and = means the pans balance.

ON ANIMAL MATH

PRINTABLE

Equations with All Four Operations: sheets 1-2

PRINTABLE

Variables and Equations: sheet 2

PRINTABLE

True or False: Multiplication & Division: sheet 2 (3rd grade), a step back

EXIT TICKET

4th Grade Multi-Step Problems and Patterns Exit Tickets: Exit Ticket 6.3

Problems and Patterns

Lesson 6.4: A letter in any place

GOAL Students find the value of a letter in any place of an equation with all four operations ($n + 156 = 400$, $350 \div m = 7$) with a related fact or the operation that undoes it, and check by putting the number back in.

About 45 minutes

★ Taught in 4th grade in some states

I can find the value of a letter in an equation.

THE LESSON

- 1 Warm up** (5 min) Cover one number of $9 \times 40 = 360$ with a sticky note, three times. Which operation finds each hidden number?
- 2 Teach** (10 min) Write $n + 156 = 400$: what plus 156 is 400? Undo the adding: $400 - 156 = 244$, so $n = 244$. Write $8 \times k = 360$: $360 \div 8 = 45$. Write $350 \div m = 7$: think $7 \times m = 350$, so $m = 50$. Write $165 = p - 48$: the letter can be on either side, and $p = 165 + 48 = 213$. Check each one by putting it back.
Say: "I ask what number makes the equation true. I undo the operation, then I check."
- 3 Try it together** (10 min) Partners write words as equations and solve them: a number times 6 is 84; a number minus 85 is 230. Then each writes an equation with the letter in a new place for the other to solve.
- 4 Practice** (15 min) Students do sheets 4-5 of Equations with All Four Operations (a letter in any place, who is right?), then sheet 1 of Variables and Equations. Sheet 3 of the first pack and sheet 4 of the second are more practice.
- 5 Exit ticket** (5 min) Students do **Exit Ticket 6.4** and circle a face beside the "I can" line. On question 1, look for $w = 223$ and $g = 70$; 73 means 75 was taken away, and 2,520 means 420 was multiplied by 6. On question 2, look for $7 \times n = 91$ and $n = 13$.

Need help? Draw a strip diagram for each equation and mark the unknown part with its letter.

Ready for more? Write three equations where $m = 12$: one with $\times$, one with $\div$ and one with $-$.

GET READY

- Fact triangle cards
- Sticky notes to cover a number
- Whiteboards and markers

WORDS TO USE

unknown: the number a letter stands for

undo: subtracting undoes adding; dividing undoes multiplying

check: put the number in place of the letter: is it true?

ASK

- What number makes the equation true?
- Which operation undoes this one?
- How can you check your answer?

WATCH FOR

Does the same operation: $n + 156 = 400$, so $n = 556$. Ask: what plus 156 is 400? Undo the adding.

Thinks the letter must come first. Read $165 = p - 48$ as "165 is the same as p take away 48".

ON ANIMAL MATH

PRINTABLE

Equations with All Four Operations: sheets 3-5

PRINTABLE

Variables and Equations: sheets 1 and 4

MAKER

Fill in the Blank Equations: Addition, Subtraction, Multiplication and Division, Range Within 100, Position Mixed (Advanced Options)

EXIT TICKET

4th Grade Multi-Step Problems and Patterns Exit Tickets: Exit Ticket 6.4



Problems and Patterns

Lesson 6.5: Number patterns from a rule

GOAL Students make number patterns from a rule (add 9, subtract 45, multiply by 2), find the rule of a pattern, and notice features the rule does not say, such as odd and even numbers taking turns, and explain why they happen.

About 45 minutes

I can make a number pattern from a rule.

THE LESSON

- 1 **Warm up** (5 min) Is $14 + 9$ odd or even? What about $23 + 9$? What happens when you add an odd number?
- 2 **Teach** (10 min) Start at 14 and add 9: 14, 23, 32, 41, 50. Circle the odd ones: odd and even take turns, because adding an odd number turns odd to even and even to odd. Start at 36 and add 20: every number is even and ends in 6, because adding 20 never changes the ones. Start at 5 and multiply by 2: 5, 10, 20, 40, 80. After the 5, every number is even.
Say: "I use the rule to get each next number. Then I look for something the rule does not say and ask why it happens."
- 3 **Try it together** (10 min) Pairs pick a start and a rule from the board (add 19, add 50, subtract 25, multiply by 3), write six numbers and trade: the partner finds the rule and a feature the rule does not say. Add the features to the class chart.
- 4 **Practice** (15 min) Students do sheets 1-3 of Number Patterns for 4th Grade (make patterns from a rule, find the rule and the missing numbers, then what the rule does not say). For a step back, sheet 2 of Arithmetic Patterns sorts odd and even sums.
- 5 **Exit ticket** (5 min) Students do **Exit Ticket 6.5** and circle a face beside the "I can" line. On question 1, look for 42, 57 and 72 and "Take turns"; "All odd" means only the start was checked. On question 2, look for 925 and 775 and "Subtract 75"; subtract 150 means the student skipped a blank.

Ready for more? Find a start and an adding rule so every number ends in 5. Then one where the ones digits go 3, 8, 3, 8.

GET READY

- Hundreds charts and two colored pencils
- Whiteboards and markers
- A class chart: what the rule does not say

WORDS TO USE

rule: what to do to get the next number

term: one number in a pattern

feature: something true about the pattern that the rule does not say

ASK

- What do you do to get the next number?
- What do you notice that the rule does not say?
- Why does that happen?

WATCH FOR

Finds the rule from only the first two numbers.

Test the rule on every pair of numbers in the pattern.

Names a feature with no reason. Ask what adding an odd number does to an odd number.

Need help? Color an adding pattern on a hundreds chart and look at the colors and the ones digits.

ON ANIMAL MATH

PRINTABLE

Number Patterns for 4th Grade: sheets 1-3

PRINTABLE

Arithmetic Patterns: sheet 2 (3rd grade), a step back

MAKER

Number Patterns: Pattern Type Mixed, Difficulty Hard

EXIT TICKET

4th Grade Multi-Step Problems and Patterns Exit Tickets: Exit Ticket 6.5

Problems and Patterns

Lesson 6.6: Shape patterns from a rule

GOAL Students build and draw shape patterns that grow by a rule, record the steps in a table, find a later step from the rule, and explain features the rule does not say.

About 45 minutes

I can make a shape pattern from a rule.

THE LESSON

- 1 Warm up** (5 min) Build a plus sign with 5 tiles. Add a tile to the end of each arm. How many now?
- 2 Teach** (10 min) Keep building the plus sign: step 1 has 5 tiles and each step adds a tile to each of its 4 arms, so 9, 13, 17. The rule: start with 5, add 4. Make a table to step 6. To find step 10 without building it, count the jumps: 9 jumps of 4 after step 1, so $5 + 36 = 41$. Every count is odd: the rule does not say so, but adding 4 to an odd number keeps it odd.
Say: "Each step adds the same number of tiles. I find a far step by counting the jumps from step 1."
- 3 Try it together** (10 min) Pairs get a rule card (start with 11, add 6) and design a shape that follows it. They fill in a table to step 6, find step 10 (65) and say why every count is odd. They can build it with the Pattern Blocks tool too.
- 4 Practice** (15 min) Students do sheet 4 of Number Patterns for 4th Grade (tile patterns with a table, step 10 and what the rule does not say), then sheets 2 and 4 of Growing Shape Patterns.
- 5 Exit ticket** (5 min) Students do **Exit Ticket 6.6** and circle a face beside the "I can" line. On question 1, look for step 3 drawn as 4 squares on 5 (9 squares) and 19 for step 8. On question 2, look for 22; 26 means 5 jumps of 4 were counted from step 1, not 4.

Ready for more? Make a shape pattern whose counts are all multiples of 4, and explain why they are.

GET READY

- Square tiles and pattern blocks for each pair
- Grid paper
- Whiteboards and markers

WORDS TO USE

step: one picture of a shape pattern

table: step numbers and counts, side by side

rule: the first step, and how many tiles each step adds

ASK

- How many tiles does each step add?
- How many jumps is it from step 1 to step 10?
- Will a step of the plus sign ever have 50 tiles? How do you know?

WATCH FOR

Counts 10 jumps to step 10. Step 1 is already there: from step 1 to step 10 is 9 jumps.

Adds the step number to the count. Point to the table: the count grows by the rule, not by the step number.

Need help? Build each step with tiles, using a new color for the tiles that step adds.

ON ANIMAL MATH

PRINTABLE

Number Patterns for 4th Grade: sheet 4

PRINTABLE

Growing Shape Patterns: sheets 2 and 4

TOOL

Pattern Blocks

EXIT TICKET

4th Grade Multi-Step Problems and Patterns Exit Tickets: Exit Ticket 6.6

Problems and Patterns

Lesson 6.7: Input-output tables

GOAL Students find the rule of an input-output table (add, subtract, multiply or divide by one number), test it on every pair, and use it to fill in missing outputs and inputs.
I can find the rule of a table and use it.

About 45 minutes
 ★ Taught in 4th grade in some states

THE LESSON

- 1 Warm up** (5 min) Guess my rule: students call out inputs and the teacher says each output (multiply by 4). After four tries, they guess the rule.
- 2 Teach** (10 min) Show a table: in 14, 25, 33, 40; out 42, 53, 61, 68. Each out is 28 more, so the rule is add 28. Test it on every pair. Then a table that starts in 4, out 12: add 8 or multiply by 3? The next pair, in 6, out 18, fits only multiply by 3. To find a missing input, undo the rule: an output of 27 comes from 9.
Say: "A rule has to work for every pair. To find an input, I undo the rule."
- 3 Try it together** (10 min) Pairs play guess my rule with secret rules of their own (multiply by 7, subtract 25), then make a table of five pairs with two blanks, one in and one out, for another pair to fill in.
- 4 Practice** (15 min) Students do sheets 2-4 of Input and Output Tables (use the rule, find the rule, then the missing numbers). Early finishers crack sheet 1 of the Secret Rule Machine.
- 5 Exit ticket** (5 min) Students do **Exit Ticket 6.7** and circle a face beside the "I can" line. On question 1, look for subtract 18, the outputs 47 and 63, and 90 as the last input ($72 + 18$). On question 2, look for 6, 8, 12 and 15.

Ready for more? Make a table where in 2, out 10 could be add 8 or multiply by 5, then add pairs so only one rule fits.

GET READY

- Input-output table cards
- Whiteboards and markers
- A rule machine drawn on the board

WORDS TO USE

input: the number that goes in

output: the number that comes out

rule: what the machine does to every input

ASK

- Does your rule work for every pair?
- Could another rule fit the first pair?
- How do you find a missing input?

WATCH FOR

Uses a rule that fits only the first pair. Test the rule on every pair before writing it.

Uses the rule to find a missing input. For an input, undo the rule: subtracting becomes adding.

Need help? Write the change from each input to its output under the table before naming the rule.

ON ANIMAL MATH

PRINTABLE

Input and Output Tables: sheets 2-4

PRINTABLE

Secret Rule Machine: Input and Output Rules: sheet 1 (3rd grade), for early finishers

EXIT TICKET

4th Grade Multi-Step Problems and Patterns Exit Tickets: Exit Ticket 6.7

Problems and Patterns

Lesson 6.8: Rules by position

GOAL Students write a rule that gives a value from its position, as an expression with n ($30 \times n$, $n + 40$), use it to find a far value, and solve pattern stories with a table.

I can write a rule for a value from its position.

About 45 minutes

★ Taught in 4th grade in some states

THE LESSON

- 1 Warm up** (5 min) A pattern goes 30, 60, 90, 120. What is the 5th number? The 10th?
- 2 Teach** (10 min) Make a table: position 1, 2, 3, 4 and value 30, 60, 90, 120. Ask how each value comes from its position: 1×30 , 2×30 , so the value at position n is $30 \times n$, and the 20th value is 600. Then position 1, 2, 3 and value 41, 42, 43: each value is 40 more than its position, $n + 40$, so the 25th value is 65.
Say: "I look across from each position to its value. The same rule has to work for every position."
- 3 Try it together** (10 min) Solve a pattern story as a class: a movie ticket costs 8 dollars. Make a table for 1 to 4 tickets, write the rule ($8 \times n$) and find what 12 tickets cost (96 dollars). Then pairs make a table for a story of their own.
- 4 Practice** (15 min) Students do sheet 5 of Variables and Equations (write the rule with n), then the pattern stories on sheet 5 of Input and Output Tables and sheet 5 of Number Patterns for 4th Grade.
- 5 Exit ticket** (5 min) Students do **Exit Ticket 6.8** and circle a face beside the "I can" line. On question 1, look for $20 \times n$ ($n \times 20$ is right too); $n + 20$ means the student looked from one value to the next, not from the position. On question 2, look for 54.

Ready for more? Find a rule where position 10 has a value of 70 and position 20 a value of 140. Is there more than one?

GET READY

- Position and value tables
- Square tiles
- Whiteboards and markers

WORDS TO USE

position: where a number is in the pattern: 1st, 2nd, 3rd

value: the number at that position

expression: a rule with n , like $30 \times n$

ASK

- How does the value come from the position?
- Does your rule work for every position?
- What is the value at position 100?

WATCH FOR

Writes the rule from one value to the next (add 30). That is the pattern's rule. Look across from the position to its value.

Puts the value in place of n . n is the position: put the position number in its place.

Need help? Build the first three positions with tiles and label each with its position.

ON ANIMAL MATH

PRINTABLE

Variables and Equations: sheet 5

PRINTABLE

Input and Output Tables: sheet 5

PRINTABLE

Number Patterns for 4th Grade: sheet 5

EXIT TICKET

4th Grade Multi-Step Problems and Patterns Exit Tickets: Exit Ticket 6.8

